Eur. Phys. J. D **42**, 467–473 (2007)
DOI: 10.1140/epjd/e2007-00044-6

THE EUROPEAN PHYSICAL JOURNAL D

Control of spontaneous emission spectra via an external coherent magnetic field in a cycle-configuration atomic medium

J.H. Li[1,2,a]

[1] Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, P.R. China
[2] Institute of Advanced Energy, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan

Received 21 November 2006 / Received in final form 16 January 2007
Published online 2nd March 2007 –

Abstract. We theoretically study the features of the spontaneous emission spectra in a coherently driven cold four-level atomic system with a cyclic configuration. It is shown that a few interesting phenomena such as spectral-line narrowing, spectral-line enhancement, and spectral-line suppression can be realized in our system. Interestingly enough, the spectral-line enhancement and suppression can be controlled just by appropriately modulating the phase, the frequency, and the intensity of an external coherent magnetic field, respectively. This investigation may find applications in high-precision spectroscopy.

PACS. 42.50.Ct Quantum description of interaction of light and matter; related experiments – 32.80.Qk Coherent control of atomic interactions with photons – 32.50.+d Fluorescence, phosphorescence (including quenching)

1 Introduction

In the last few decades there has been intensive interest in the study of spontaneous emission originating from the interaction of the atomic system with the environmental mode. The theoretical approach in the control and modification of spontaneous emission is widely discussed [1–16]. The potential applications for such a spontaneous-emission control cover from lasing without inversion [17–22], high-precision spectroscopy and magnetometry [23–25], transparent high-index materials [26,27], quantum information and computing [28–30], and so on. As it is well-known, for atoms in free space, atomic coherence and quantum interference are the basic phenomena for efficient control of spontaneous emission [31,32]. It has been shown that the control of spontaneous emission can be achieved just by putting atoms into different environment, such as in free-space, photonic crystals and in optical cavities, which have different densities of electromagnetic modes interacting with atoms. An alternative method of controlling spontaneous emission is to couple atoms with external coherent fields. The quenching of spontaneous emission in an open V-type atom was investigated in reference [6]. Phase-dependent effects in spontaneous emission spectra in a four-level atom were proposed in reference [9] and for an atom near the edge of a photonic band gap in reference [11]. Recently, Paspalakis and Knight presented a phase control scheme in a four-level atom driven by two lasers of the same frequencies in the presence of the spontaneously generated quantum interference, where the relative phase of the two lasers was used to obtain partial cancellation, extreme linewidth narrowing, and total cancellation in the spontaneous emission spectrum [7]. This spontaneously generated quantum interference can lead to the appearance of ultranarrow spectral lines [7,33], gain without inversion [21], atomic population trapping in excited levels [34], phase-dependent line shapes [9,15,35], and pulse-preserving propagation in dissipative media [36,37]. However, it should be noted that the existence of this quantum interference, which is usually referred to as spontaneously generated coherence (SGC) or vacuum-induced coherence (VIC), requires that two close-lying levels be near-degenerate and that the atomic dipole moments be nonorthogonal when the atom is placed in free space. Unfortunately, it is very difficult, if not impossible, to find a real atomic system with SGC to experimentally realize these phenomena because the rigorous conditions of near-degenerate levels and nonorthogonal dipole matrix elements cannot be simultaneously satisfied. As a result, few experiments have been performed to achieve these interesting phenomena based on SGC. More recently, Wu and his coworkers studied the spontaneous-emission properties of a coherently driven four-level atom, and showed a few interesting phenomena such as fluorescence quenching, spectral-line narrowing, spectral-line enhancement, and spectral-line elimination. They also pointed out that these phenomena could be observed in the experiment since the rigorous condition of near-degenerate levels with nonorthogonal dipole

[a] e-mail: huajia_li@163.com

moments was not required [38]. In the following research [39], Li et al. studied a different four-level atomic model and arrived at similar conclusions. To the best of our knowledge, no further theoretical or experimental work has been carried out to study such atomic spontaneous decay properties in a four-level atomic system with a cyclic configuration via an external coherent magnetic field, which motivate the current work.

In this paper we put forward a new scheme for the four-level atom with cyclic or Δ-type transitions in which we can efficiently control the spontaneous emission via varying the phase, the frequency, and the intensity of an external coherent magnetic field, and the nature of this field is determined by the level structure (could be infrared, microwave, radio-frequency (RF) etc., depending upon the structure of the atomic levels). Of particular interest is the application of an external coherent magnetic field, as the RF (or microwave) source is more readily available and easier to control in comparison with an extra laser field, and this is the situation considered in this paper. In our scheme, the quantum coherence between two lower ground levels is induced by an external coherent magnetic field instead of the sharing of the vacuum modes by the two transitions. The proposed scheme requires three driving fields but is more convenient in its experimental realization.

The remainder of this paper is organized into three parts as follows. In Section 2, the model is presented. The basic dynamics equations of motion, and their solution for the spontaneous emission spectra are derived. In Section 3, we analyze our results and discuss in detail the influence of the phase, the frequency, and the intensity of the external coherent magnetic field on the spontaneous emission spectra. A possible experimental realization of our scheme is also proposed. Finally, we conclude with a brief summary in Section 4.

2 Model and solution

Consider a medium of four-level atoms with three ground states $|1\rangle$, $|2\rangle$, $|4\rangle$ and one excited states $|3\rangle$ as depicted in Figure 1. A coherent probe laser with carrier frequency ω_p and Rabi frequency $2\Omega_p$ drives the transition $|1\rangle \leftrightarrow |3\rangle$. At the same time, a coherent coupling laser with carrier frequency ω_c and Rabi frequency $2\Omega_c$ is used to couple the transition $|2\rangle \leftrightarrow |3\rangle$. The transition $|2\rangle \leftrightarrow |1\rangle$ is electric dipole forbidden transition while magnetic dipole allowed, and we apply an external control magnetic field with carrier frequency ω_d and Lamor frequency $2\Omega_d$ to the optical transition $|2\rangle \leftrightarrow |1\rangle$, and the nature of this field is determined by the level structure (could be infrared, RF, microwave, etc., depending upon the structure of the atomic levels [40]). While the transition from the state $|3\rangle$ to the state $|4\rangle$ is assumed to be coupled by the vacuum modes in the free space. The interaction of driven transitions with the vacuum modes is neglected. With the rotating-wave approximation and the electro-dipole approximation, the whole Hamiltonian describing the atom-field interaction for the system under study in the Schrödinger's picture,

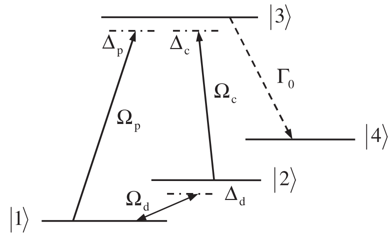

Fig. 1. Schematic diagram of four-level atoms in a coherent medium interacting with a probe laser with Rabi frequency $2\Omega_p$, a coupling laser with Rabi frequency $2\Omega_c$, and an external coherent magnetic field with Lamor frequency $2\Omega_d$ (the transition $|1\rangle \leftrightarrow |2\rangle$ is electric dipole forbidden transition while magnetic dipole allowed). The atomic states are labelled as $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$, respectively. The transitions $|1\rangle \rightarrow |2\rangle \rightarrow |3\rangle \rightarrow |1\rangle$ owns a cyclic configuration. Δ_p, Δ_c, and Δ_d are the frequency detunings of the corresponding probe, coupling, and coherent magnetic fields, see text for details.

is given by

$$\begin{aligned} H = & \sum_{j=1}^{4} \hbar\omega_j |j\rangle\langle j| + \sum_k \hbar\omega_k b_k^+ b_k \\ & + \hbar\Big(\Omega_p e^{-i\omega_p t}|3\rangle\langle 1| + \Omega_c e^{-i\omega_c t}|3\rangle\langle 2| + \Omega_d e^{-i\omega_d t}|2\rangle\langle 1| \\ & + \sum_k g_{4,k} b_k |3\rangle\langle 4| + h.c.\Big), \end{aligned} \quad (1)$$

where the symbol $h.c.$ means the hermitian conjugate, the quantities Ω_p, Ω_c, and Ω_d are one-half Rabi an Lamor frequencies for the relevant driven transitions, i.e., $\Omega_p = \mu_{31}E_p/(2\hbar)$, $\Omega_c = \mu_{32}E_c/(2\hbar)$ and $\Omega_d = \mu_{21}E_d/(2\hbar)$ with $\mu_{mn} = \boldsymbol{\mu}_{mn} \cdot \boldsymbol{e}_L$ ($\boldsymbol{e}_L$ is the unit polarization vector of the corresponding laser field; m, n = 1–3) denoting the dipole moment for the transition between levels $|m\rangle$ and $|n\rangle$, and $E_j^{(a)} = \hbar\omega_j$ (j = 1–4) is the energy of the atomic state $|j\rangle$. $b_k^\dagger$ and b_k are respectively the creation and annihilation operators for the kth vacuum mode with frequency ω_k; k here represents both the momentum vector and the polarization of the emitted photon. $g_{4,k}$ stands for the coupling constant between the kth vacuum mode and the atomic transition $|3\rangle \leftrightarrow |4\rangle$. For the sake of simplicity, in following analysis we will take $\omega_1 = 0$ for the ground state $|1\rangle$ as the energy origin. According to the spirit of reference [41] via choosing the proper free Hamiltonian, turning to the interaction picture, with the assumption of $\hbar = 1$, the free and interaction Hamiltonian can be respectively rewritten as follows

$$H_0 = (\omega_p - \omega_c)\,|2\rangle\langle 2| + \omega_p|3\rangle\langle 3| + \omega_4|4\rangle\langle 4| + \sum_k \omega_k b_k^+ b_k,$$

$$\begin{aligned} H_I = & (\Delta_p - \Delta_c)\,|2\rangle\langle 2| + \Delta_p|3\rangle\langle 3| \\ & + \Big(\Omega_p|3\rangle\langle 1| + \Omega_c|3\rangle\langle 2| + \Omega_d|2\rangle\langle 1| \\ & + \sum_k g_{4,k} b_k e^{-i(\Delta_p - \delta_k)t}|3\rangle\langle 4| + h.c.\Big), \end{aligned} \quad (2)$$

where the frequency detunings of three driving fields and the vacuum modes are defined respectively by

$\varDelta_p = \omega_{31} - \omega_p = \omega_3 - \omega_p$ ($\omega_1 = 0$), $\varDelta_c = \omega_{32} - \omega_c$, $\varDelta_d = \omega_{21} - \omega_d$, and $\delta_k = \omega_{34} - \omega_k$, as shown in Figure 1. In the above derivation process, we have assumed that the carrier frequencies of the three fields satisfy $\omega_p = \omega_c + \omega_d$ for the sake of simplification, the relationship $\varDelta_p = \varDelta_c + \varDelta_d$ can be obtained.

The wave function of the atomic system, at a specific time t, can be expanded in terms of the bare-state eigenvectors such that

$$|\Psi(t)\rangle = [a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle]\,|\{0\}\rangle + \sum_k a_{4,k}(t)|4\rangle|1_k\rangle, \tag{3}$$

where $a_j(t)$ ($j = 1$–4) stands for the time-dependent probability amplitude of the atomic state $|j\rangle$. $|\{0\}\rangle$ represents the absence of photons in all vacuum modes, and $|1_k\rangle$ indicates that there is one photon in the kth vacuum mode.

Making use of the well-known Schrödinger equation in the interaction picture $i\partial|\Psi(t)\rangle/\partial t = H_I|\Psi(t)\rangle$ and performing the Weisskopf-Wigner approximation [42], the coupled equations of motion for the probability amplitude evolution of the atomic wave functions can be readily obtained as

$$\frac{\partial a_1(t)}{\partial t} = -i\Omega_p^* a_3(t) - i\Omega_d^* a_2(t), \tag{4a}$$

$$\frac{\partial a_2(t)}{\partial t} = -i(\varDelta_p - \varDelta_c)\,a_2(t) - i\Omega_c^* a_3(t) - i\Omega_d a_1(t), \tag{4b}$$

$$\frac{\partial a_3(t)}{\partial t} = -i\left(\varDelta_p - i\frac{\Gamma_0}{2}\right)a_3(t) - i\Omega_p a_1(t) - i\Omega_c a_2(t), \tag{4c}$$

$$\frac{\partial a_{4,k}(t)}{\partial t} = -ig_{4,k}^* e^{i(\varDelta_p - \delta_k)t} a_3(t), \tag{4d}$$

where $\Gamma_0 = 2\pi|g_{4,k}|^2 D(\omega_k)$ is the spontaneous-decay rate from level $|3\rangle$ to level $|4\rangle$, and $D(\omega_k)$ is the vacuum-mode density at frequency ω_k in the free space.

Carrying out the Laplace transformations $\tilde{a}_j(s) = \int_0^\infty e^{-st} a_j(t)dt$ [s is the time Laplace transform variable] for equations (4a–4c) and integrating equation (4d) with respect to t', we have the results

$$s\tilde{a}_1(s) - a_1(0) = -i\Omega_p^*\tilde{a}_3(s) - i\Omega_d^*\tilde{a}_2(s), \tag{5a}$$

$$s\tilde{a}_2(s) - a_2(0) = -iw_2\tilde{a}_2(s) - i\Omega_c^*\tilde{a}_3(s) - i\Omega_d\tilde{a}_1(s), \tag{5b}$$

$$s\tilde{a}_3(s) - a_3(0) = -iw_3\tilde{a}_3(s) - i\Omega_p\tilde{a}_1(s) - i\Omega_c\tilde{a}_2(s), \tag{5c}$$

$$a_{4,k}(t) = -ig_{4,k}^*\int_0^t e^{iw_4 t'} a_3(t')dt', \tag{5d}$$

where we have introduced the definitions $w_2 = \varDelta_p - \varDelta_c$, $w_3 = \varDelta_p - i\Gamma_0/2$, and $w_4 = \varDelta_p - \delta_k$, respectively. $a_j(0)$ ($j = 1$–3) is the probability amplitude at the initial time $t = 0$.

Equations (5a–5c) can be solved directly in terms of $a_1(0)$, $a_2(0)$, and $a_3(0)$, therefore the solution to the probability amplitude $\tilde{a}_3(s)$ can be found as

$$\tilde{a}_3(s) = \frac{f_3(s)a_3(0) - f_2(s)a_2(0) - f_1(s)a_1(0)}{f(s)}, \tag{6}$$

with

$$\begin{aligned} f_3(s) &= |\Omega_d|^2 + s(s + iw_2), \\ f_2(s) &= i\Omega_c s + \Omega_p\Omega_d^*, \\ f_1(s) &= i\Omega_p(s + iw_2) + \Omega_c\Omega_d, \\ f(s) &= s(s + iw_2)(s + iw_3) + |\Omega_d|^2(s + iw_3) + s|\Omega_c|^2 \\ &\quad + |\Omega_p|^2(s + iw_2) - i\Omega_p\Omega_c^*\Omega_d^* - i\Omega_p^*\Omega_c\Omega_d. \end{aligned}$$

As it is well-known, the spontaneous emission spectra is the Fourier transformation of $\langle E^-(t+\tau)E^+(t)\rangle_{t\to\infty}$, and can be expressed as the form $S(\delta_k) = (\Gamma_0/2\pi|g_{4,k}|^2)\,|a_{4,k}(t\to\infty)|^2$ for our studied atomic system. According to the above equation (5d), we have $a_{4,k}(t\to\infty) = -ig_{4,k}^*\int_0^\infty e^{iw_4 t'} a_3(t')dt' = -ig_{4,k}^*\tilde{a}_3(s = -iw_4)$. And substituting the expression (6) of $\tilde{a}_3(s)$ into the expression of $a_{4,k}(t\to\infty)$, we can obtain

$$\begin{aligned} S(\delta_k) &= \frac{\Gamma_0}{2\pi|g_{4,k}|^2}|a_{4,k}(t\to\infty)|^2 = \frac{\Gamma_0}{2\pi}|\tilde{a}_3(s = -iw_4)|^2 \\ &= \frac{\Gamma_0}{2\pi}\left|\frac{f_3(\delta_k)a_3(0) - f_2(\delta_k)a_2(0) - f_1(\delta_k)a_1(0)}{f(\delta_k)}\right|^2, \end{aligned} \tag{7}$$

where the coefficients are given by

$$\begin{aligned} f_3(\delta_k) &= |\Omega_d|^2 - w_4(w_4 - w_2), \\ f_2(\delta_k) &= \Omega_c w_4 + \Omega_p\Omega_d^*, \\ f_1(\delta_k) &= \Omega_p(w_4 - w_2) + \Omega_c\Omega_d, \\ f(\delta_k) &= w_4(w_4 - w_2)(w_4 - w_3) - |\Omega_d|^2(w_4 - w_3) \\ &\quad - w_4|\Omega_c|^2 - |\Omega_p|^2(w_4 - w_2) - \Omega_p\Omega_c^*\Omega_d^* - \Omega_p^*\Omega_c\Omega_d. \end{aligned}$$

Equation (7) is the main results of the present study. In order to account for the effect of the phase of the external coherent magnetic field on the spontaneous emission spectra, we replace Ω_d by $|\Omega_d|e^{i\Phi}$. Again we assume Ω_p and Ω_c to be real, i.e., $\Omega_p = |\Omega_p|$ and $\Omega_c = |\Omega_c|$ in the following analysis. Under the conditions $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, and $\varDelta_p = \varDelta_c = 0$, we then have $w_2 = 0$, $w_3 = -i\Gamma_0/2$, and $w_4 = -\delta_k$ so that the spontaneous emission spectra $S(\delta_k)$ in equation (7) can be explicitly reduced into the following form

$$S(\delta_k, \Phi) = \frac{\Gamma_0}{2\pi}\frac{(|\Omega_c||\Omega_d| - |\Omega_p|\delta_k)^2 + 2|\Omega_p||\Omega_c||\Omega_d|\delta_k(1 - \cos\Phi)}{\left[\delta_k^3 - \left(|\Omega_p|^2 + |\Omega_c|^2 + |\Omega_d|^2\right)\delta_k + 2|\Omega_p||\Omega_c||\Omega_d|\cos\Phi\right]^2 + \frac{1}{4}\Gamma_0^2\left(\delta_k^2 - |\Omega_d|^2\right)^2}. \tag{8}$$

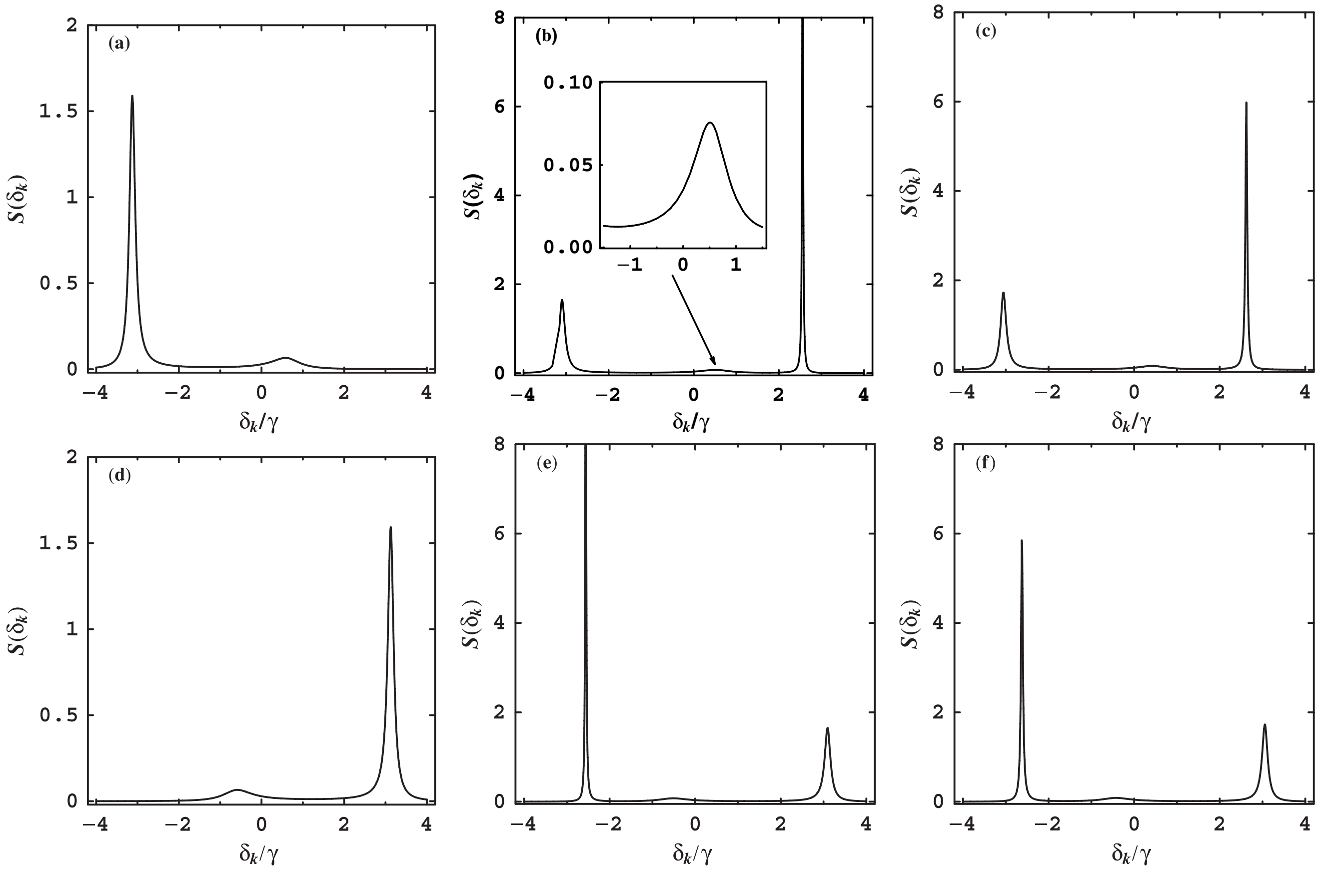

Fig. 2. Spontaneous emission spectra $S(\delta_k)$ (in units of γ^{-1}) for $\Omega_p = \gamma$, $\Omega_c = \gamma$, $|\Omega_d| = 2.5\gamma$, $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, $\Gamma_0 = \gamma$, $\Delta_p = 0$, $\Delta_c = 0$ and $\Delta_d = 0$. (a) $\Phi = 0$; (b) $\Phi = \pi/6$; (c) $\Phi = \pi/4$; (d) $\Phi = \pi$; (e) $\Phi = 7\pi/6$; and (f) $\Phi = 5\pi/4$. The inset displays an enlargement of the central peak for clarity.

Under another conditions $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, $\Delta_p = 0$, $\Delta_c = -\Delta_d$, and $\Phi = 0$, we then have $w_2 = \Delta_d$, $w_3 = -i\Gamma_0/2$, and $w_4 = -\delta_k$ so that the spontaneous emission spectra $S(\delta_k)$ in equation (7) can be also reduced into the following form

$$S(\delta_k, \Delta_d) = \frac{\Gamma_0}{2\pi} \frac{\left[|\Omega_c|\,|\Omega_d| - |\Omega_p|\,(\delta_k + \Delta_d)\right]^2}{G}, \quad (9)$$

with

$$G = \Big[\delta_k^3 + \Delta_d \delta_k^2 - \left(|\Omega_p|^2 + |\Omega_c|^2 + |\Omega_d|^2\right)\delta_k - \Delta_d\,|\Omega_p|^2 + 2\,|\Omega_p|\,|\Omega_c|\,|\Omega_d|\Big]^2 + \frac{1}{4}\Gamma_0^2\left(\delta_k^2 + \Delta_d\delta_k - |\Omega_d|^2\right)^2.$$

3 Results and discussion

In this section, we present a few numerical results about the spontaneous emission spectra $S(\delta_k)$. All parameters used in the calculations are scaled by γ, which should be in the order of MHz for rubidium or sodium atoms. First of all, we will analyze how the phase Φ of the external coherent magnetic field modifies the spontaneous emission spectra $S(\delta_k)$ via the numerical calculations based on equation (7). In Figure 2, we plot the spontaneous emission spectra $S(\delta_k)$ versus the detuning δ_k for different phases Φ under the initial condition of $a_1(0) = 1$ and $a_2(0) = a_3(0) = 0$ when the three driving fields is tuned to the resonant interaction with the corresponding atomic transitions. When the phase Φ is adjusted from 0 to $5\pi/4$, the spontaneous emission spectra have a conversion from double-peak structure to three-peak structure. Specifically, for the case that $\Phi = 0$ (see Fig. 2a), a high side peak and an extremely suppressed central peak occur. An alternative side peak is completely suppressed. For the case that $\Phi = \pi/6$, a pair of side peaks and one almost completely suppressed central peak can be observed. The height of left side peak is raised somewhat with respect to the case that $\Phi = 0$, while new right side peak with ultranarrow line can be greatly enhanced (see Fig. 2b). For the case that $\Phi = \pi/4$, left side peak and central peak change little. The height of right side peak decreases rapidly (see Fig. 2c). It is straightforward to show from Figure 2 that we can obtain the new spectral line corresponding to $\pi + \Phi$ from the old spectral line corresponding to Φ just by replacing δ_k with $-\delta_k$ (see Figs. 2d, 2e, and 2f), as can be easily seen from equation (8), i.e., $S(\delta_k, \pi + \Phi) = S(-\delta_k, \Phi)$. On the basis of the above analysis, we can see that by adjusting the phases of the external coherent magnetic field, such as $\Phi = \pi/6$ and $\Phi = 7\pi/6$ (see Figs. 2b and 2e), large enhancement of the

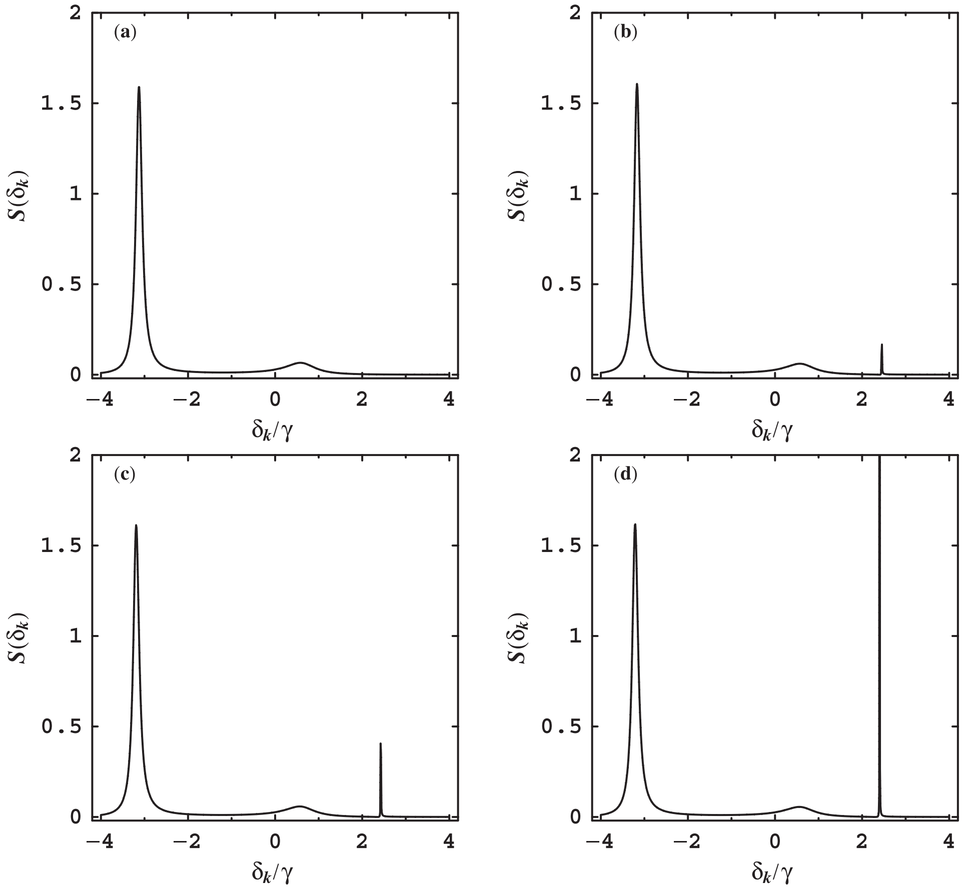

Fig. 3. Spontaneous emission spectra $S(\delta_k)$ (in units of γ^{-1}) for $\Omega_p = \gamma$, $\Omega_c = \gamma$, $|\Omega_d| = 2.5\gamma$, $\Phi = 0$, $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, $\Gamma_0 = \gamma$, $\Delta_p = 0$, and $\Delta_c = -\Delta_d$. (a) $\Delta_d = 0$; (b) $\Delta_d = 0.1\gamma$; (c) $\Delta_d = 0.15\gamma$; and (d) $\Delta_d = 0.2\gamma$.

ultranarrow line can be achieved and the behavior of the spontaneous emission spectra can be controlled efficiently as well.

In Figure 3, we plot the spontaneous emission spectra $S(\delta_k)$ versus the detuning δ_k based on equation (9) by modulating frequencies Δ_d of the external coherent magnetic field under the same initial condition as above except that $\Phi = 0$ when the probe driving field is tuned to the resonant interaction with the atomic transition $|3\rangle \leftrightarrow |1\rangle$. It is clearly shown that, when the external coherent magnetic field is tuned to level $|2\rangle$ (i.e., $\Delta_d = 0$ in Fig. 3a), one side peak and an extremely suppressed central peak can be observed. Interestingly, for a small frequency detuning of the external coherent magnetic field, i.e., $\Delta_d = 0.1\gamma$, a new right ultranarrow side line appears (see Figs. 3b and 3c). With the increase of Δ_d to further higher value, right ultranarrow side line can be greatly enhanced (see Fig. 3d).

Figure 4 shows the effect of external coherent magnetic field intensity on the spontaneous emission spectra $S(\delta_k)$ when the probe, coupling and external coherent magnetic fields are respectively tuned to the resonant interaction with the atomic transitions $|1\rangle \leftrightarrow |3\rangle$, $|2\rangle \leftrightarrow |3\rangle$, and $|1\rangle \leftrightarrow |2\rangle$. As can be seen, the change of the magnetic field intensity affects appreciably both the width and the height of the spectral lines. With gradual increase of the intensity of the external coherent magnetic field, we can see that the height of left side peak increases and its width becomes narrower, while the height of right side peak decreases slowly. Specifically, for the case that no external coherent magnetic field exists (i.e., $|\Omega_d| = 0$), the spontaneous emission spectra exhibit symmetrical double-peak structure with equal height and normal linewidth restricted by spontaneous decay rate Γ (see Fig. 4a). In contrast, when the external coherent magnetic field is applied, for the case that $|\Omega_d| = 1.5\gamma$ and 2.5γ, the position of left enhanced side peak keeps away from $\delta_k = 0$, while the position of right suppressed side peak closes with $\delta_k = 0$. When the external coherent magnetic field intensity continues to increase (e.g., $|\Omega_d| = 3.5\gamma$ in Fig. 3d), greatly enhanced side peak with narrow width can be obtained.

Before ending this section, let us briefly discuss the possible experimental realization of our proposed scheme by means of alkali-metal atoms, appropriate diode lasers and microwave source. Specifically, we consider for instance the cold atoms ^{87}Rb–D_2 line (nuclear spin $I = 3/2$) as a possible candidate. The designated states can be chosen as follows: $|1\rangle = |5S_{1/2}, F = 1, m_F = 0\rangle$, $|2\rangle = |5S_{1/2}, F = 2, m_F = 0\rangle$, $|3\rangle = |5P_{3/2}, F = 2, m_F = 1\rangle$, and $|4\rangle = |5S_{1/2}, F = 2, m_F = 2\rangle$, respectively. In this case, the two coherent laser radiations at ω_p and ω_c are sidebands created by modulating the frequency of a

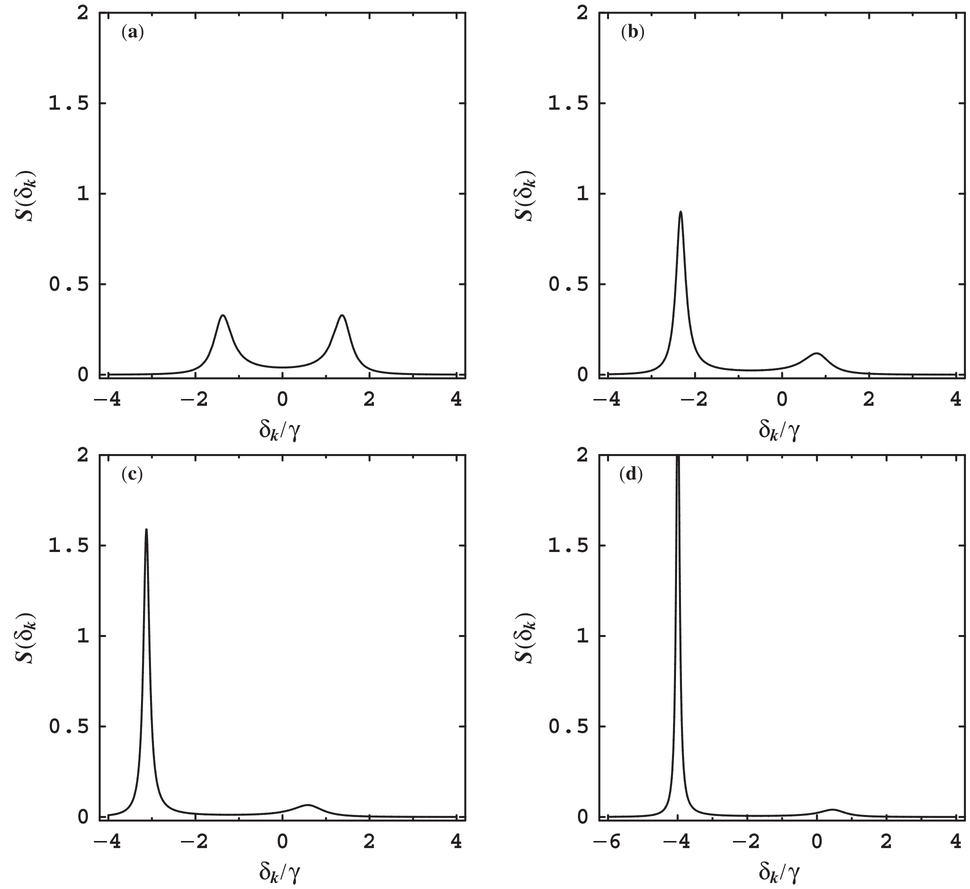

Fig. 4. Spontaneous emission spectra $S(\delta_k)$ (in units of γ^{-1}) for $\Omega_p = \gamma$, $\Omega_c = \gamma$, $\Phi = 0$, $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, $\Gamma_0 = \gamma$, $\Delta_p = 0$, $\Delta_c = 0$ and $\Delta_d = 0$. (a) $|\Omega_d| = 0$; (b) $|\Omega_d| = 1.5\gamma$; (c) $|\Omega_d| = 2.5\gamma$; and (d) $|\Omega_d| = 3.5\gamma$.

laser tuned to the $5S_{1/2}$ to $5P_{3/2}$ transition (D_2 line at 780.2 nm). A microwave field, which can be created by means of a cavity as illustrated in reference [43], drives the magnetic dipole transition between $|1\rangle = |5S_{1/2}, F = 1, m_F = 0\rangle$ and $|2\rangle = |5S_{1/2}, F = 2, m_F = 0\rangle$ with the hyperfine splitting frequency $\omega_{21} = 6.84$ GHz. While the transition from the excited level $|3\rangle = |5P_{3/2}, F = 2, m_F = 1\rangle$ to the metastable level $|4\rangle = |5S_{1/2}, F = 2, m_F = 2\rangle$ can be coupled by the vacuum modes in the free space. Moreover, in order to eliminate the Doppler broadening effect, atoms should be trapped and cooled by the magneto-optical trap (MOT) technique.

4 Conclusions

In summary, we have theoretically investigated the spontaneous emission spectra of a coherently driven four-level atomic system by means of the probe field, the coupling driving field, and the external coherent magnetic field. The results clearly show that, by properly adjusting the phase, the frequency detuning, and the intensity of the external coherent magnetic field, we can observe a few interesting phenomena in the spontaneous emission spectra, such as spectral-line narrowing, spectral-line enhancement, and spectral-line suppression. According to our analysis, these interesting phenomena should be observable in realistic experiments by using cold Rb or Na atoms, appropriate diode lasers and microwave source or radio-frequency field.

The research is supported in part by the National Natural Science Foundation of China under Grant Nos. 10575040, 90503010 and 10634060 and by National Basic Research Program of China under Contract No. 2005CB724508. The author would like to thank Professor Wu Ying for helpful discussion and his encouragement.

References

1. P. Zhou, S. Swain, Phys. Rev. Lett. **78**, 832 (1997)
2. S. John, T. Quang, Phys. Rev. Lett. **78**, 1888 (1997)
3. P. Zhou, S. Swain, Phys. Rev. A **55**, 772 (1997)
4. C.H. Keitel, P.L. Knight, L.M. Narducci, M.O. Scully, Opt. Commun. **118**, 143 (1995)
5. H. Lee, P. Polynkin, M.O. Scully, S.Y. Zhu, Phys. Rev. A **55**, 4454 (1997)
6. E. Paspalakis, C.H. Keitel, P.L. Knight, Phys. Rev. A **58**, 4868 (1998)
7. E. Paspalakis, P.L. Knight, Phys. Rev. Lett. **81**, 293 (1998)
8. M.A. Antón, O.G. Calderón, F. Carreño, Phys. Rev. A **72**, 023809 (2005); X.M. Hu, J.S. Peng, J. Phys. B **33**, 921 (2000)

9. F. Ghafoor, S.Y. Zhu, M.S. Zubairy, Phys. Rev. A **62**, 013811 (2000)
10. S.Y. Zhu, M.O. Scully, Phys. Rev. Lett. **76**, 388 (1996)
11. S.Y. Zhu, H. Chen, H. Huang, Phys. Rev. Lett. **79**, 205 (1997)
12. H. Huang, S.Y. Zhu, M.S. Zubairy, Phys. Rev. A **55**, 744 (1997)
13. T. Hong, C. Cramer, W. Nagourney, E.N. Fortson, Phys. Rev. Lett. **94**, 050801 (2005)
14. F. Ghafoor, S. Qamar, M.S. Zubairy, Phys. Rev. A **65**, 043819 (2002)
15. M.A.G. Martinez, P.R. Herczfeld, C. Samuels, L.M. Narducci, C.H. Keitel, Phys. Rev. A **55**, 4483 (1997)
16. G.X. Li, F.L. Li, S.Y. Zhu, Phys. Rev. A **64**, 013819 (2001)
17. G.S. Agarwal, Phys. Rev. Lett. **84**, 5500 (2000)
18. G.S. Agarwal, Phys. Rev. Lett. **67**, 980 (1991)
19. A.S. Zibrov, M.D. Lukin, D.E. Nikonov, L. Hollberg, M.O. Scully, V.L. Velichansky, H.G. Robinson, Phys. Rev. Lett. **75**, 1499 (1995)
20. S.E. Harris, Phys. Rev. Lett. **62**, 1033 (1989)
21. E. Paspalakis, S.Q. Gong, P.L. Knight, Opt. Commun. **152**, 293 (1998)
22. J.Y. Gao, C. Guo, X.Z. Guo, G.X. Jin, Q.W. Wang, J. Zhao, H.Z. Zhang, Y. Jiang, D.Z. Wang, D.M. Jiang, Opt. Commun. **93**, 323 (1992)
23. M.O. Scully, M. Fleischhauer, Phys. Rev. Lett. **69**, 1360 (1992)
24. M. Fleischhauer, A.B. Matsko, M.O. Scully, Phys. Rev. A **62**, 013808 (2000)
25. T. Hong, C. Cramer, W. Nagourney, E.N. Fortson, Phys. Rev. Lett. **94**, 050801 (2005)
26. M.O. Scully, Phys. Rev. Lett. **67**, 1855 (1991)
27. M. Fleischhauer, C.H. Keitel, M.O. Scully, C. Su, B.T. Ulrich, S.Y. Zhu, Phys. Rev. A **46**, 1468 (1992)
28. C.H. Bennett, D.P. Divincenzo, Nature **404**, 247 (2000)
29. D. Petrosyan, Y.P. Malakyan, Phys. Rev. A **70**, 023822 (2004)
30. M. Paternostro, M.S. Kim, P.L. Knight, Phys. Rev. A **71**, 022311 (2005)
31. E. Arimondo, in *Progress in Optics*, edited by E. Wolf (Elsevier, Amsterdam, 1996)
32. G.S. Agarwal, *Quantum Optics* (Springer-Verlag, Berlin, 1974), p. 68
33. P. Zhou, S. Swain, Phys. Rev. Lett. **77**, 3995 (1996); C.H. Keitel, Phys. Rev. Lett. **83**, 1307 (1999)
34. S. Menon, G.S. Agarwal, Phys. Rev. A **61**, 013807 (2000)
35. S. Menon, G.S. Agarwal, Phys. Rev. A **57**, 4014 (1998)
36. E. Paspalakis, N.J. Kylstra, P.L. Knight, Phys. Rev. Lett. **82**, 2079 (1999); E. Paspalakis, N.J. Kylstra, P.L. Knight, Phys. Rev. A **61**, 045802 (2000)
37. A. Fountoulakis, A.F. Terzis, E. Paspalakis, Phys. Rev. A **73**, 033811 (2006)
38. J.H. Wu, A.J. Li, Y. Ding, Y.C. Zhao, J.Y. Gao, Phys. Rev. A **72**, 023802 (2005)
39. A.J. Li, J.Y. Gao, J.H. Wu, L. Wang, J. Phys. B: At. Mol. Opt. Phys. **38**, 3815 (2005)
40. A. Godone, F. Levi, J. Vanier, Phys. Rev. A **59**, R12 (1999); G.S. Agarwal, T.N. Dey, S. Menon, Phys. Rev. A **64**, 053809 (2001)
41. Y. Wu, J. Saldana, Y. Zhu, Phys. Rev. A **67**, 013811 (2003); Y. Wu, X. Yang, Phys. Rev. A **70**, 053818 (2004); Y. Wu, X. Yang, Phys. Rev. A **71**, 053806 (2005)
42. V. Weisskopf, E.P. Wigner, Z. Phys. **54**, 63 (1930); S.M. Barnett, P.M. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, Oxford, 1997)
43. J. Vanier, A. Godone, F. Levi, Phys. Rev. A **58**, 2345 (1998), and references therein